

Geo372
Vertiefung GIScience

Working with terrain models

Herbstsemester

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Last week

- We looked at **spatial interpolation** – crucially we saw that **spatial autocorrelation** must be present
- We looked at a range of interpolators (local/ global; exact/ approximate; smooth/ abrupt; etc...)
- In our example, **best results** were for **ordinary kriging**, followed by **IDW** with “default” parameters (in terms of **RMSE**)
- However, **most techniques** gave similar results – other, **qualitative**, features of the **surfaces** needed to be compared
- I introduced **regionalised variable theory** to help us look at a more theoretical approach through **geostatistics**

Learning objectives

You will:

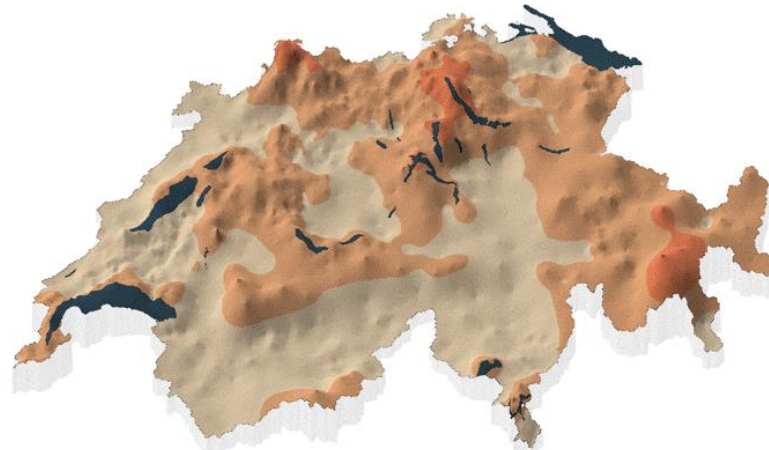
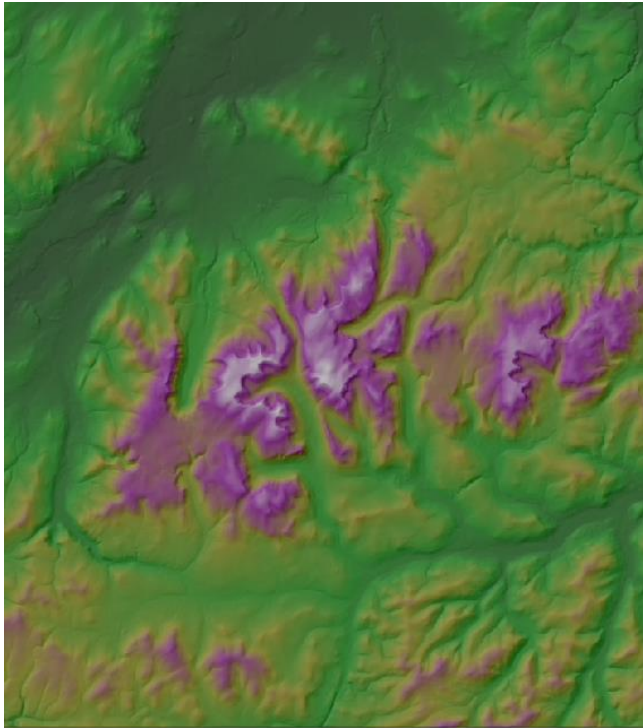
- be able to define ***gradient, aspect, plan*** and ***profile curvature, flow direction, flow accumulation, sinks and catchments***;
- be able to describe and comment on ***algorithms*** to calculate the above variables;
- be able to give examples of ***physical properties*** where the above variables are ***important***;
- be able to explain how **resolution** and **data quality** may influence these properties; and
- critically discuss **applications of derivatives** of terrain models

Today's outline

- Different ways of **modelling surfaces** – what conceptual model do you have for terrain data?
- Applications of **terrain data**
- Definitions: **slope, curvature and hydrology**
- Why **terrain matters**
- **Algorithms** for terrain models
- Raster **resolution** and slope
- Examples of terrain in **spatial analysis**

Examples of surfaces

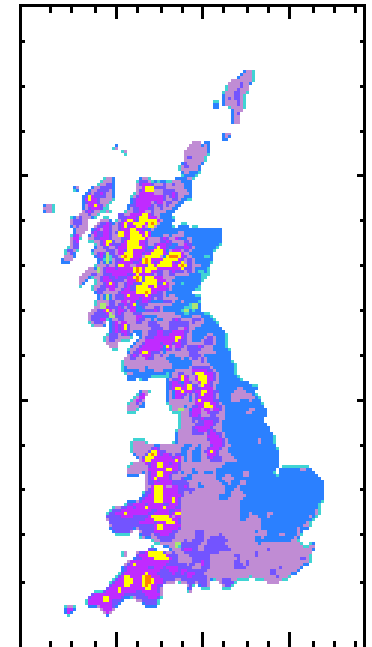
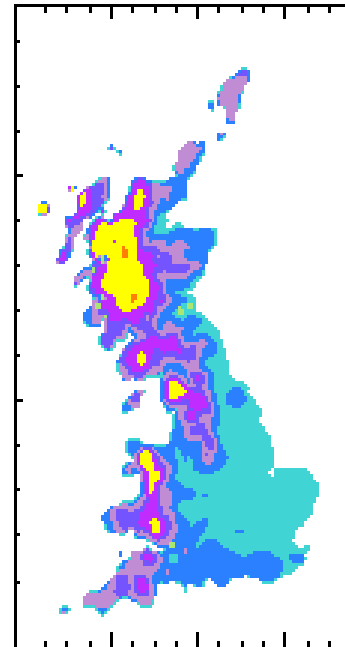
2000



mittlere Monatsmiete 4-Zimmerwohnung [CHF]

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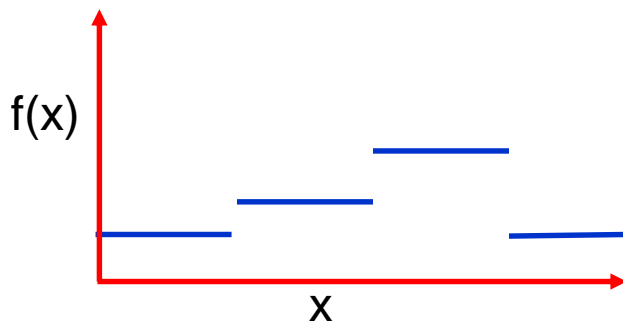
Daten: Wiest & Partner
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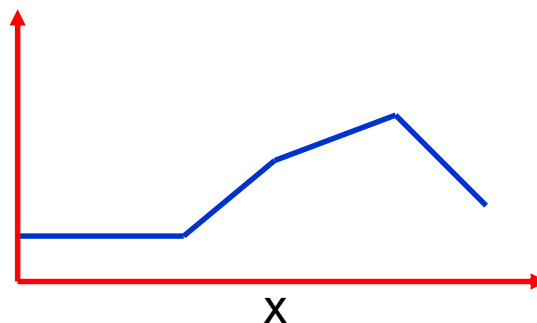
Everything I show today can be applied to any surface with **continuity** – but we will **focus on terrain**

Modelling surfaces

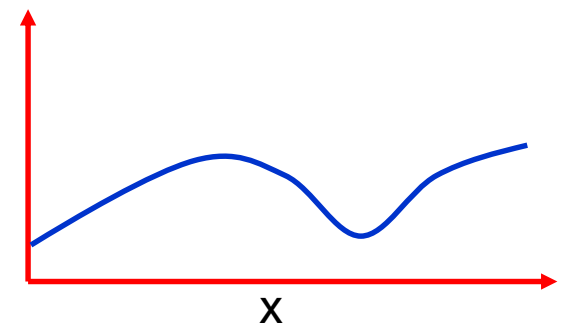
- **Continuity** – implies a value at every point
- Different levels of continuity (can be defined mathematically)



Step-wise continuous



Continuous with abrupt change in slope

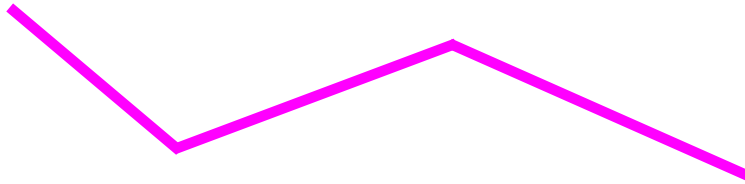


Continuous with continuous rate of change of slope

You should understand what these properties mean in terms of the **value** and **derivatives** of the function $f(x)$ for each of these graphs. Can we calculate $f'(x)$ everywhere?

Formal definitions of continuity

- G_0 -> **Values** everywhere are continuous but derivatives are not
-> $f(x)$ continuous



- G_1 -> **First derivatives** are also continuous
-> $f(x), f'(x)$ continuous



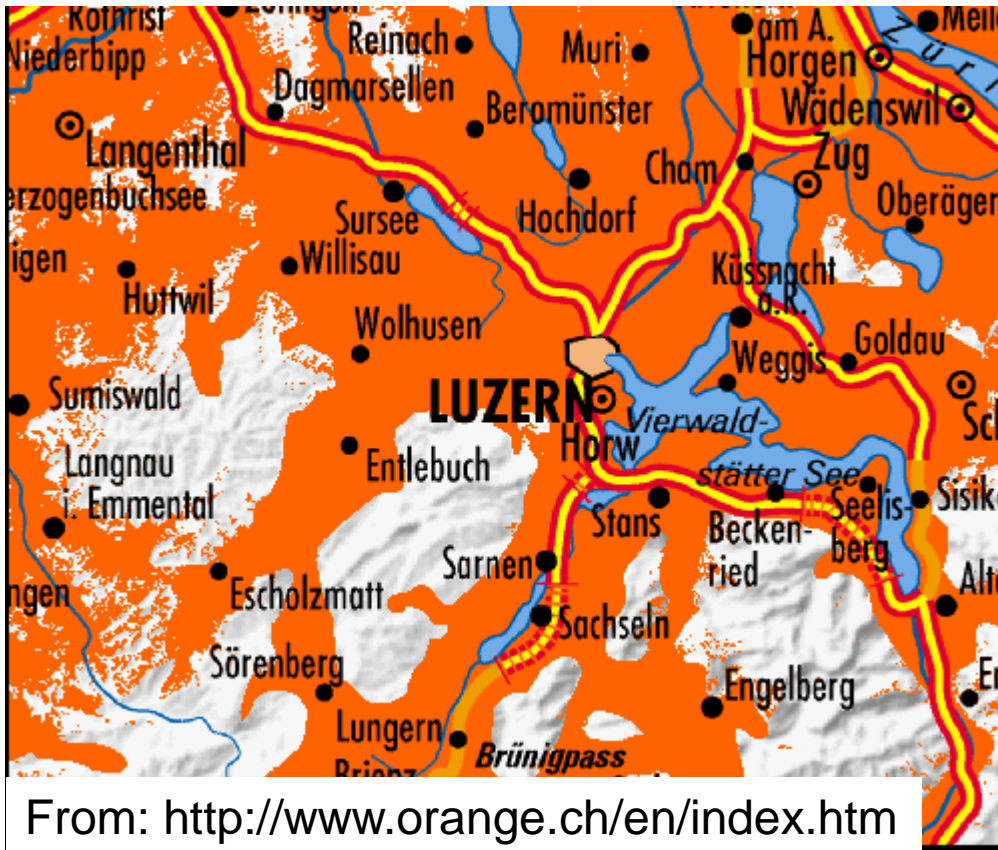
- G_2 -> **Second derivatives** are continuous
-> $f(x), f'(x), f''(x)$ continuous

Harder to draw – think about the properties

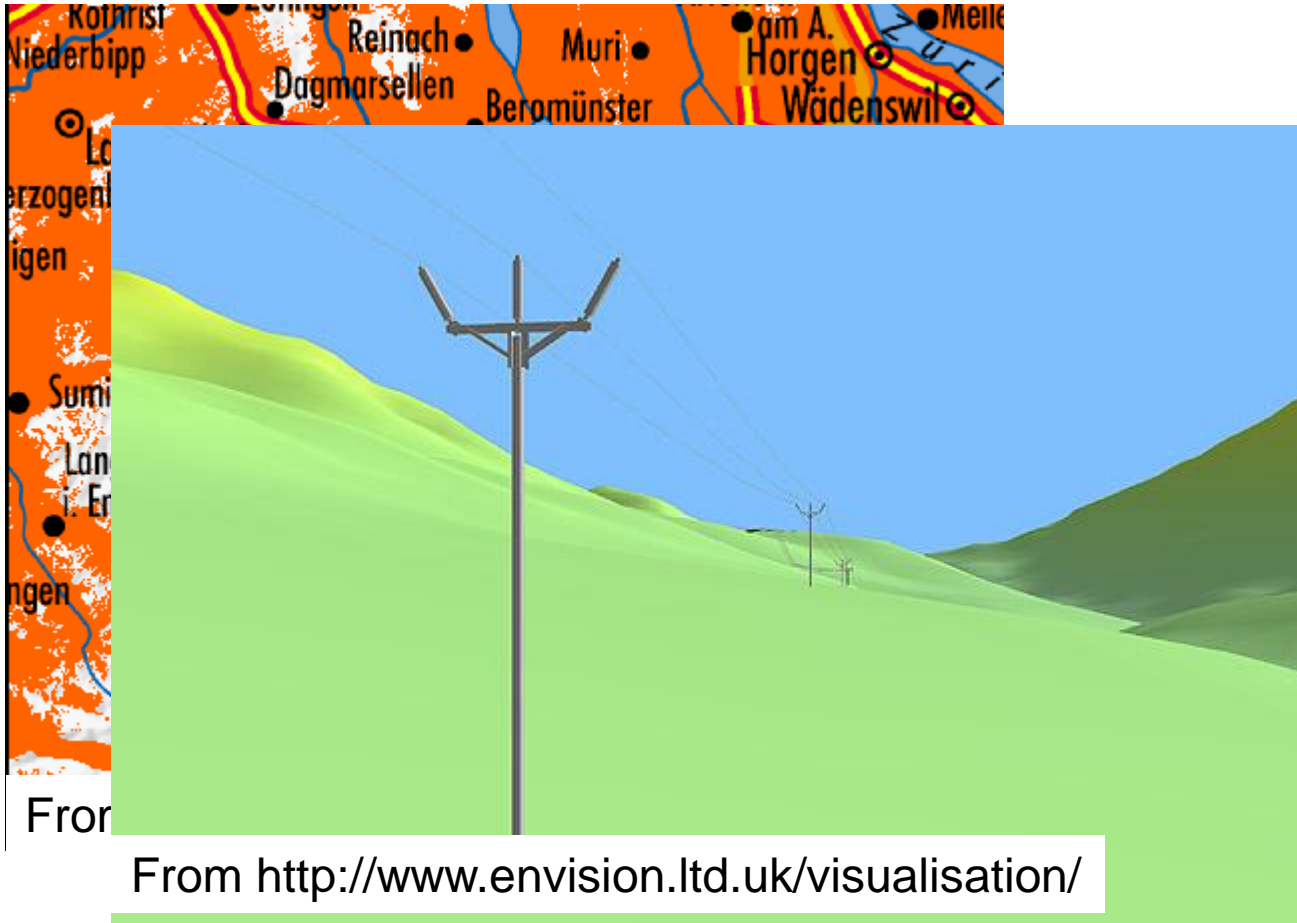
Terrain data

- Most common surface analysed is that of the **surface of the earth**
- Terrain is almost always represented as having a **single height for any point** – what cannot be represented like this?
- Many different terrain data sets exist – I won't talk about how they are **collected** or **derived**
- We want to know how we can use them in **spatial analysis!**

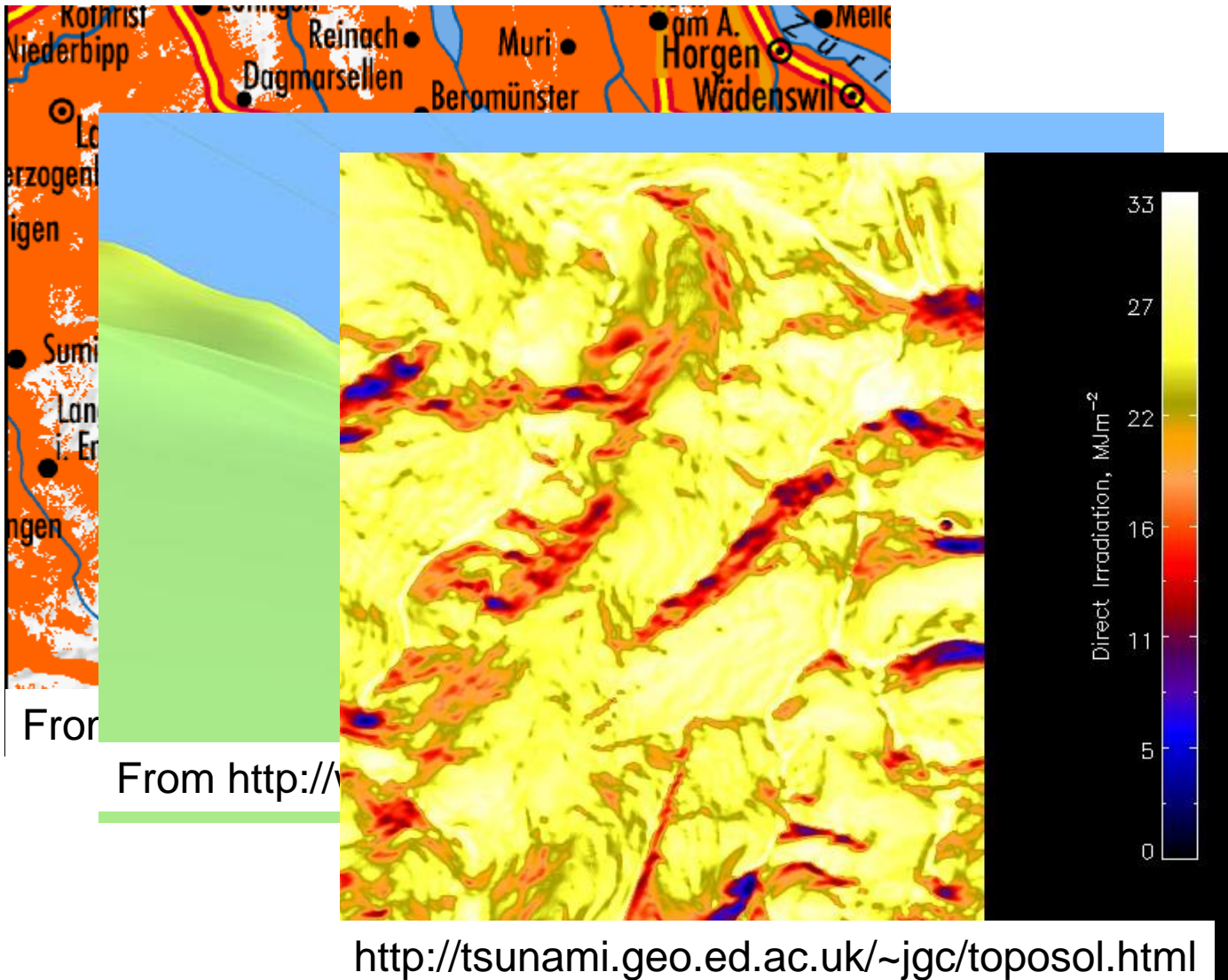
Some applications of terrain data



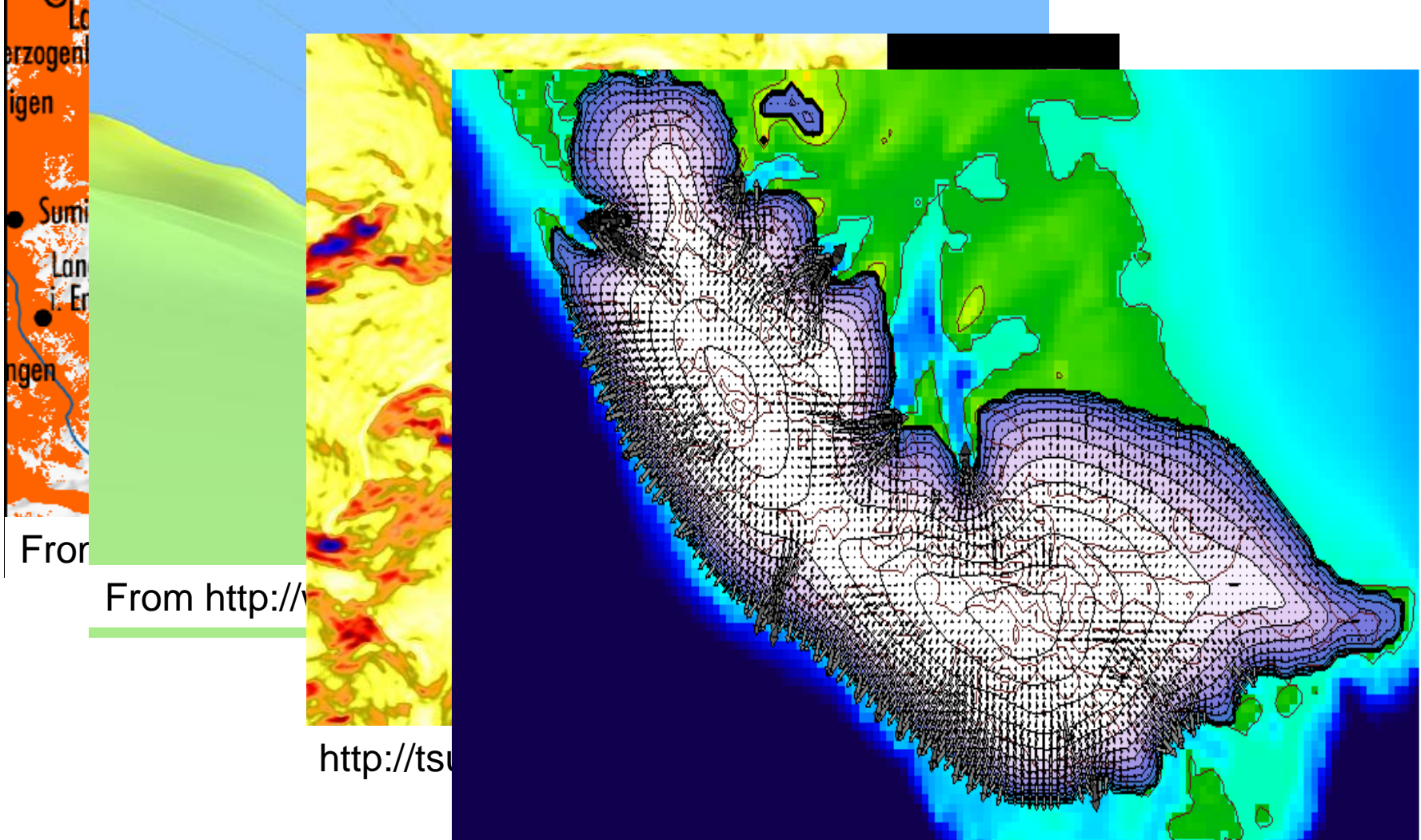
Some applications of terrain data



Some applications of terrain data



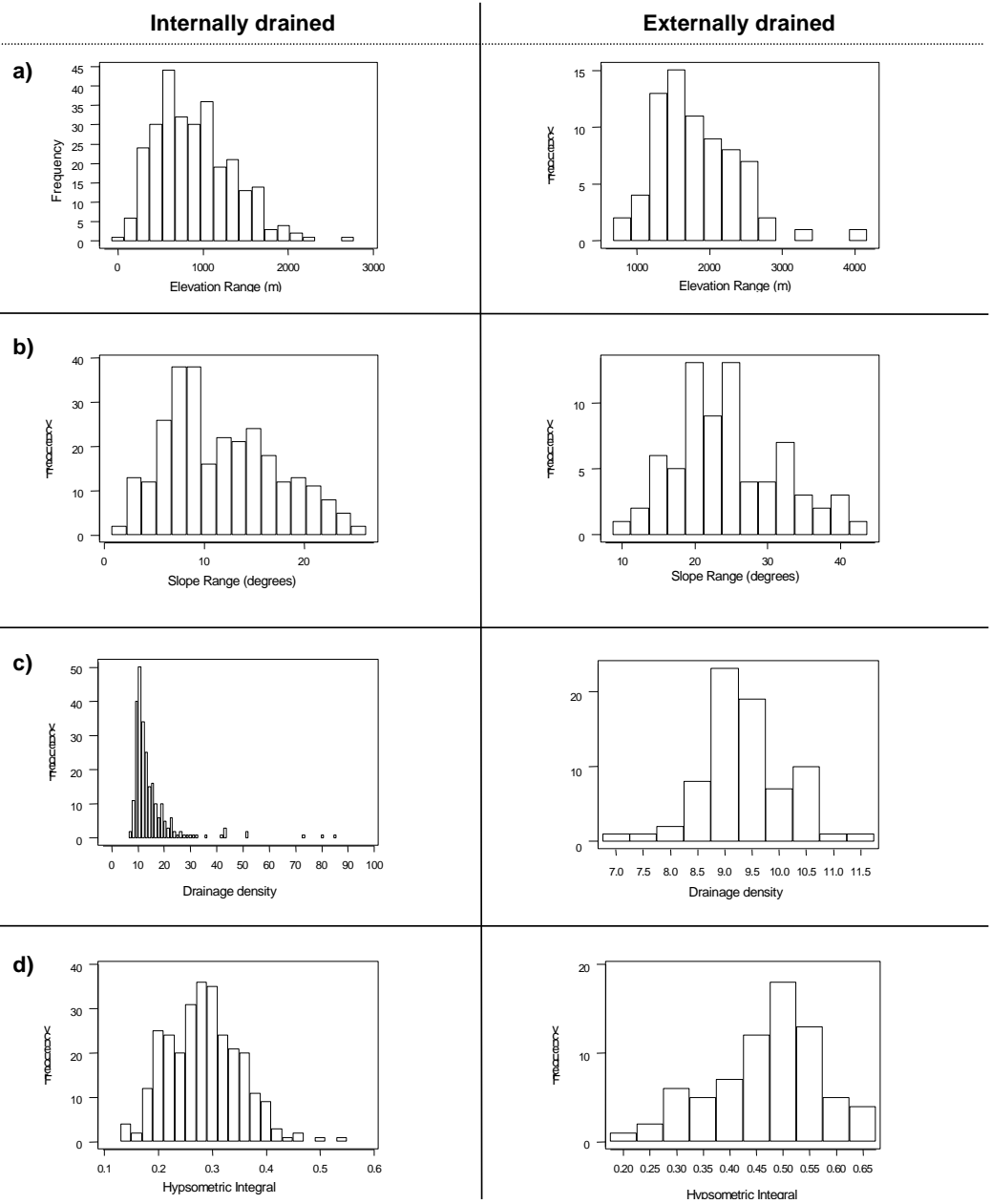
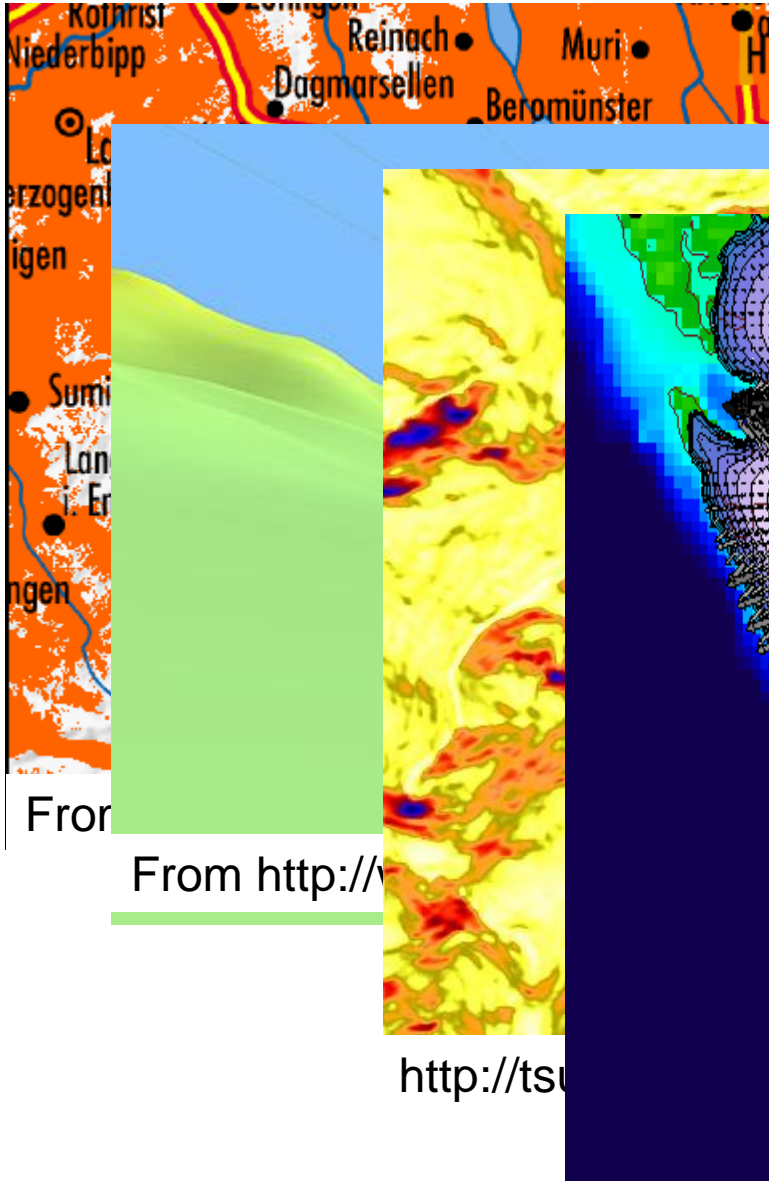
Some applications of terrain data



From <http://>

<http://tsu>

Some applicat



From a Master's thesis by Holmes looking at internal drainage and geomorphometry in Tibet

Definitions: slope

- **Slope** is formally described by a **plane** at a **tangent** to a point on the surface
- Slope has two components:
 - **Gradient:** the maximum rate of change of the elevation of the plane (the angle that the plane makes with a horizontal surface)
 - **Aspect:** the direction of the plane with respect to some arbitrary zero (usually north!)
- Note that many people/GIS (including me 😊) mix-up the terms **slope** and **gradient** – properly slope describes the properties of the **tangent plane to the surface** (represented by a *vector*)

Definitions: curvature

Curvature is the **rate of change of slope** and Evans (1980) suggested two important components, both of which can **be convex, concave or planar**

Profile curvature

- Rate of change of gradient
 - **Convex:** increase in slope gradient
 - **Concave:** decrease in gradient downslope
 - **Planar:** constant gradient

Plan curvature

- Rate of change of aspect
 - **Convex:** aspect lines diverge downslope
 - **Concave:** aspect lines converge downslope
 - **Planar:** aspect constant

Definitions: hydrology

- **Flow direction:** Inferred direction of water flow given a surface representation
- **Flow accumulation:** Total flow accumulated (e.g. the sum) at a given point on a surface – equivalent to the **drainage area upstream** of a given point
- **Sink:** A **local minimum** (depression) in a surface representation
- **Catchment:** The **drainage area** associated with a point, often defined with respect to a stream junction

Why does terrain matter? (1)

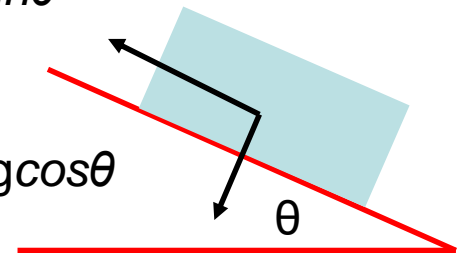
Because of gravity – no gradient = no skiing and no avalanches



At the simplest of levels...

$$F_{\parallel} = mg \sin \theta$$

$$F_{\perp} = mg \cos \theta$$



Why does terrain matter? (2)

- Relationship between aspect and house prices
 - Do you live on the

Pfnüselküste or the **Goldküste**?

- Incoming radiation for a **given point** at a **given time** is a function of the **gradient** and **aspect** of that point - south facing slopes are sunnier!
- The following equation says this (S_0 is the received extraterrestrial radiation, and T_0 the atmospheric transmittance...

solar zenith angle (how high the sun is)

solar azimuth (the direction of the sun)

$$I = [\cos \theta_0 \cos \beta + \sin \theta_0 \sin \beta \cos(\phi_0 - A)] S_0 \times \exp(-T_0 / \cos \theta_0)$$

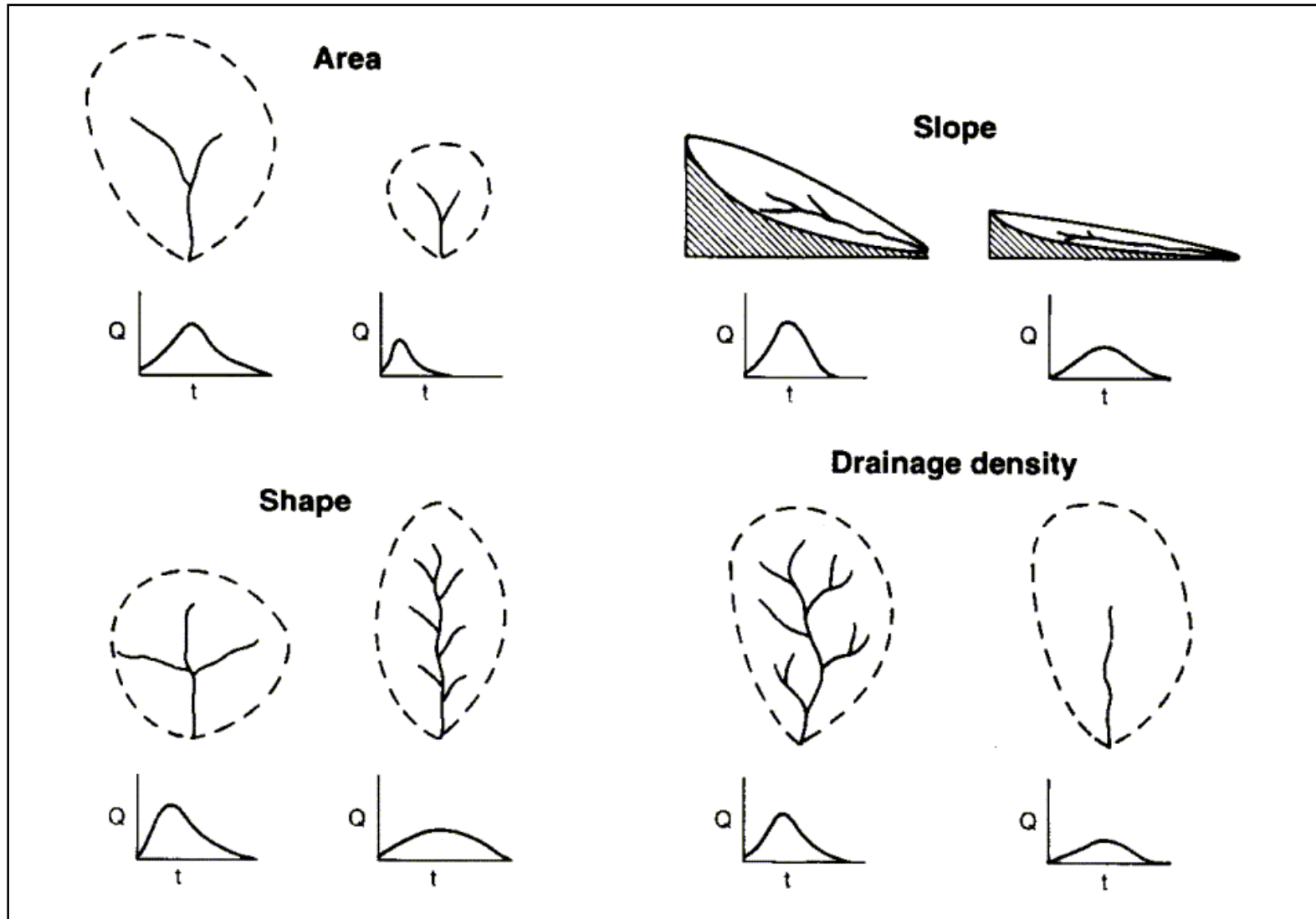
gradient

aspect

Why does terrain matter? (3)

- Many physical processes in the environment are dependent on the properties of terrain
- Topography **shapes** and is **shaped by hydrological** processes
- Catchments are a **key unit** in understanding natural processes and anthropogenic influences on the landscape, e.g.:
 - Prediction of **stream formation** and **erosion**
 - **Sediment** and **nutrient transport** in landscape
 - **Flood forecasting** e.g. through snow melt, thunder storms etc...

Linking properties



Influence of basin characteristics on hydrograph

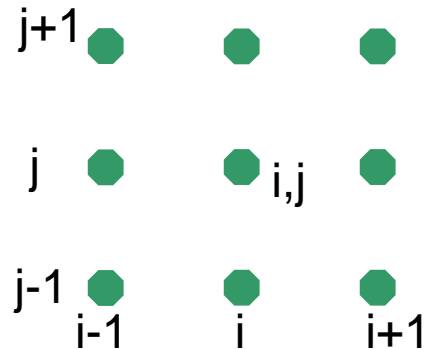
(from http://www.geog.ox.ac.uk/students/teaching/pg1_ith/)

Raster algorithms for slope and curvature

- We are going to focus on algorithms based on representing elevation values on a **regular (typically rectangular) grid**
- Point density is thus **not adapted** to complexity of relief
- Excessive data for **simple** terrain – **insufficient** data for complex terrain
- Algorithms often **simple** to implement and typically use moving window

¹Other data structures and approaches exist (e.g. TINs) but less popular in commercial GIS

Slope algorithm 1: Steepest drop



Calculate steepest drop from point/cell –
A classic example of a **focal function**

$$\textit{Gradient} = \max \left\{ \begin{array}{l} \varphi(i) \frac{z_{i,j} - z_{i+a,j+b}}{\lambda} \\ a=-1,0,1 \\ b=-1,0,1 \end{array} \right\}$$

$\varphi(i) = 1$ for N,S,E,W neighbours

$= 1/\sqrt{2}$ for NE,SE,NW,SW

λ is resolution

- Aspect is simply **direction of steepest drop**
- Only 8 possible aspects
- **Local errors** in terrain can mean **large errors**
- Considers **centre cell**

Slope algorithm 2: D8

- **Steepest drop algorithm** is commonly used to calculate flow direction in hydrology (where all flow is passed to one cell)
- Algorithm is **identical to steepest drop**, but retain only direction with a **different representation**
- Direction is defined as 2^n , where $0 \leq n \leq 7$ are cell addresses giving values 1, 2, 4, 8, ...128
- Why might these values be better than 0, 45, 90, 135,...?

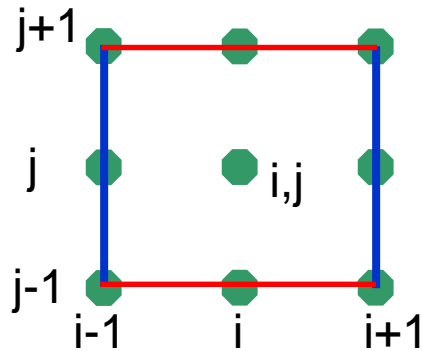
z_6	z_7	z_8
z_5	z_9	z_1
z_4	z_3	z_2

Cell addressing
(z_i)

32	64	128
16		1
8	4	2

Flow directions

Slope algorithm 3: Finite differencing



Many approaches use some sort of **finite differencing**. This example uses all 8 neighbours of a cell (but not the cell value itself)

$$\frac{\partial z}{\partial x} = [(z_{i+1,j+1} + 2z_{i+1,j} + z_{i+1,j-1}) - (z_{i-1,j+1} + 2z_{i-1,j} + z_{i-1,j-1})] / 8\partial x$$

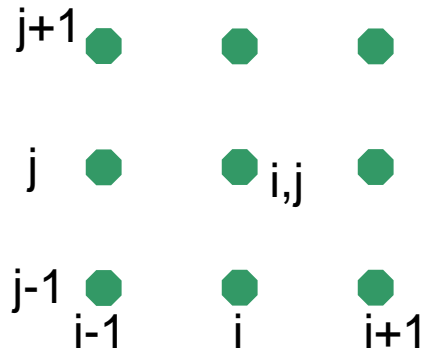
$$\frac{\partial z}{\partial y} = [(z_{i+1,j+1} + 2z_{i,j+1} + z_{i-1,j+1}) - (z_{i+1,j-1} + 2z_{i,j-1} + z_{i-1,j-1})] / 8\partial y$$

Gradient and slope are thus

$$gradient = \tan^{-1} \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}$$

$$aspect = -\tan^{-1} \left(\frac{\left(\frac{\partial z}{\partial x}\right)}{\left(\frac{\partial z}{\partial y}\right)} \right)$$

Slope algorithm 4: Fitting local quadratic



Wood and Evans both fit a **quadratic surface** to the points of the form:

$$z = ax^2 + by^2 + cxy + dx + ey + f$$

Thus

$$\delta z / \delta x = 2ax + cy + d$$

$$\delta z / \delta y = 2by + cx + e$$

and if we use a local co-ordinate system then we can set

$$x, y = 0 \text{ and so } \delta z / \delta x = d; \delta z / \delta y = e$$

The quadratic **does not** pass through all the points.

The parameters are found by matrix algebra.

Uses all 9 cells

Comparing algorithms

- A number of authors have compared algorithms for calculating slope in rasters (e.g. Skidmore, 1989; Dunn and Hickey, 1998)
- Skidmore compared **6 methods**: 2 based on **maximum drop**; a 2nd order finite difference method for a **2x2 window**; 3rd order finite differences for **3x3 window**; 2 **linear regression** models based on **3x3 window** (similar to quadratic method)
- Compared methods with '**true**' **value** calculated by drawing tangents perpendicular to contour through cell...
- **Statistically significant correlation** between gradient and aspect for all methods
- **Maximum drop** methods found to give **poorest** estimates (slope too steep, and only 8 possible aspects)
- Methods considering **3x3** windows **best** since all adjacent cells contribute...
- **BUT** Dunn and Hickey argue that a method based on maximum downhill slope retains the cell itself and better represents local variability (and doesn't smooth slope values)
- Argue that this variability is important in **processes** such as **erosion**

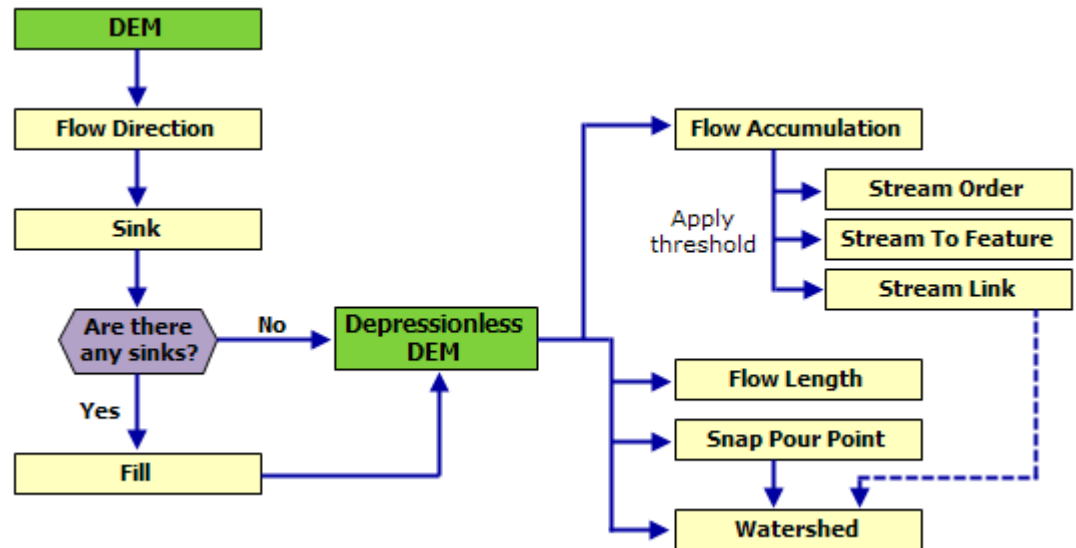
Different algorithms are appropriate for different purposes – though 3x3 windows usually give best results...

Deriving curvature from rasters

- Wood's and Evan's methods allow calculation of curvature from the quadratic **directly**
- ArcGIS uses a 4th order polynomial fitted to the points with 9 parameters ([link](#))
- ArcGIS thus uses a **different** implicit surface to find the slope – so slope and curvature are derived from different surfaces

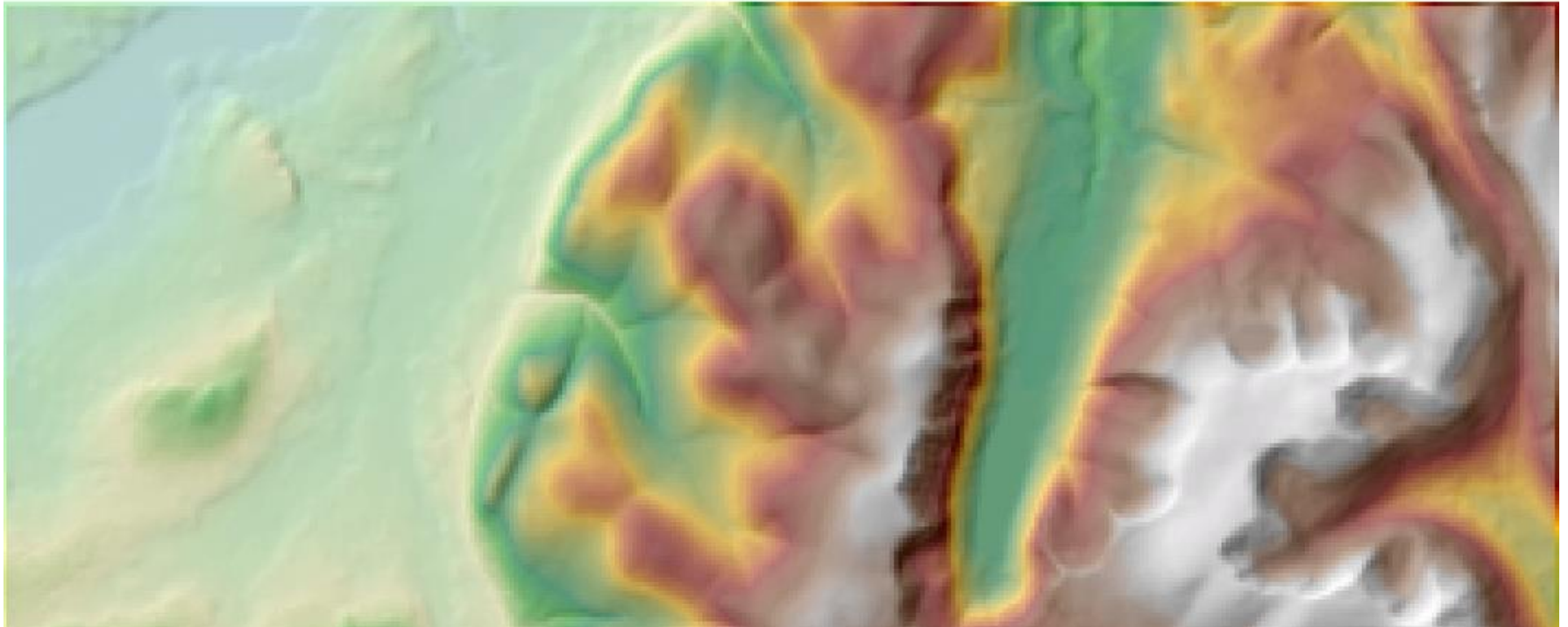
Hydrology in GIS

- In GIS we typically **simplify hydrological systems** by **assuming overland flow** (no water absorbed by soils) and **channelled flow** (flow converges)
- Typically we derive **flow direction**, check for **sinks**, calculate **flow accumulation** and derive **channel network** and **catchments**



Schematic of hydrological tools in [ArcGIS](#)

Example topography – Glen Feshie



From: http://www.gla.ac.uk/medicalgenetics/feshie.htm#Glen_Feshie

Back to D8

- Recall flow directions can **only have** intervals of 45°
- Where steepest drop occurs in **multiple directions**, we can either:
 - **Assign both** these directions to cell (by summing directions – ArcGIS does this)
 - **Assign 1st** direction to cell
 - Flag direction as **undefined**
- **D8**, despite its simplicity, is the standard method in many GIS

Modelling channel networks

- Having derived flow directions, we can derive **channel networks**
- Many **different approaches** exist to modelling channel networks
 - Those based around **focal operators** e.g. local maxima/ minima, convexities and concavities
 - “**Hydrologically-based**” algorithms using our flow directions
 - Other approaches derived from areas such as **image processing** (you have seen such methods in remote sensing)
- We will look at the **second approach**

Calculating flow accumulation

- We will use **flow direction** as a starting point
- Flow accumulation is then calculated as the **sum of upstream elements** draining to a cell
 - For every cell **count** how many **neighbours drain** to it
 - For **each of these cells** do the same
 - Repeat until all cells are either on the **boundary** of the grid or have **no upstream cells**
- When we calculate the flow accumulation for a cell we can also (if we **flag** the contributing cells) calculate the **catchment boundary** for that cell)
- If we choose some **pour point** in the channel network, we can calculate the **catchment upstream** of that point (which defines the watershed)

Flow accumulation example

32	64	128
16		1
8	4	2

Flow direction key

32	16	16	16	16	16
64	32	16	32	16	16
64	64	32	64	64	32
64	32	32	32	32	32
64	32	16	32	32	32
64	16	32	32	32	16

FlowAcc = 2+1+1+2+1

35	13	12	2	1	0
10	9	0	8	4	0
9	4	2	2	1	0
7	0	3	1	1	0
2	3	1	2	0	0
1	0	0	0	1	0

Associated watershed

Potential problems with flow accumulation

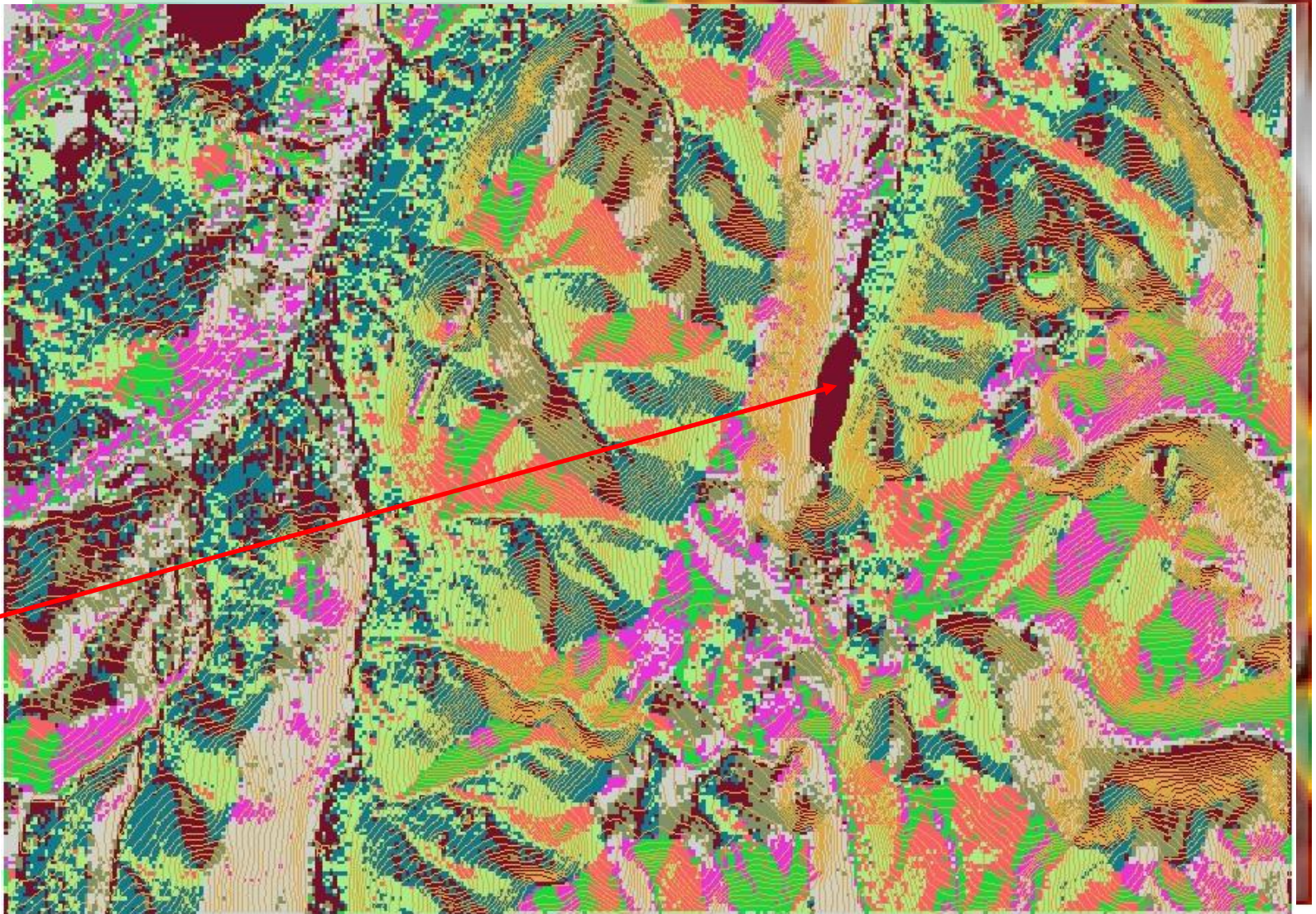
- If flow directions **converge** on a single cell, with no outflow, then flow accumulation network is **broken**
- Such locations are local minima known as **sinks** and are generally considered to be **undesirable artefacts** of the terrain model
- When might this **not be the case?**

NB: In ArcGIS, **undefined flow directions** are also treated as sinks

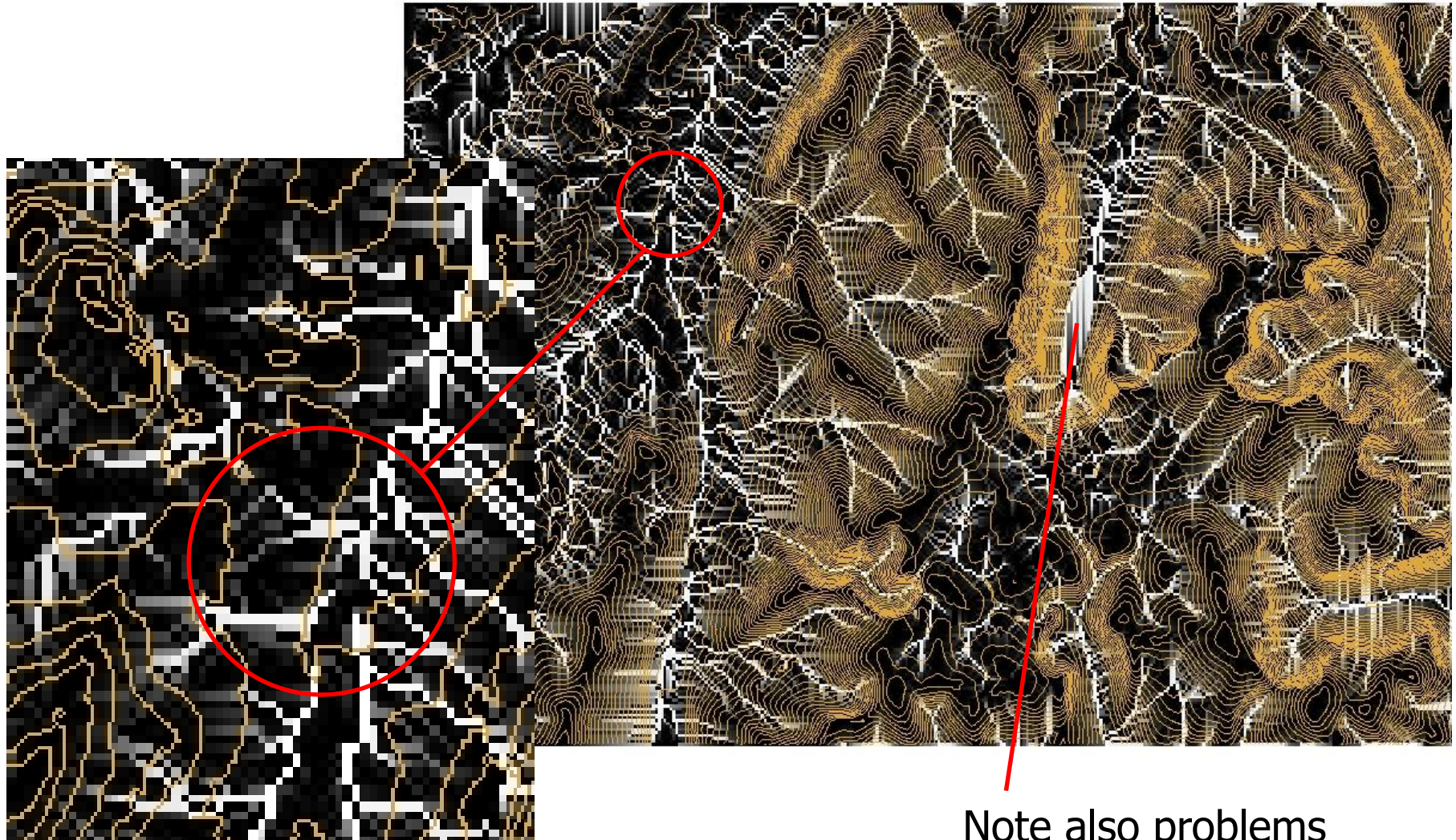
D8 results on big grid...

59, not 8
unique
values
because
of sinks

Lake



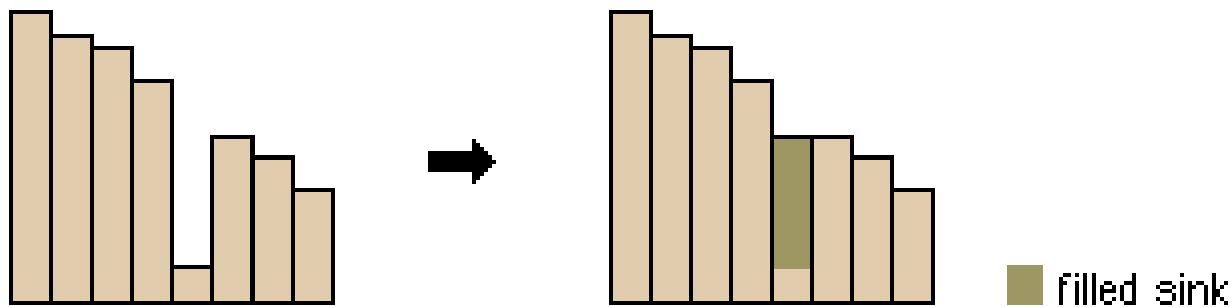
... and resulting flow accumulation network broken by sinks



Note also problems
in lake

Dealing with sinks

- Two basic strategies to remove sinks:
 - 1) Find the nearest adjacent cell with elevation the same as or lower than the sink – **hard code a flow direction to it** (*radial search*)
 - 2) **Increase the elevation of the sink** to that of a neighbouring cell – check neighbouring cell then drains (*sink filling*) (ArcGIS approach)

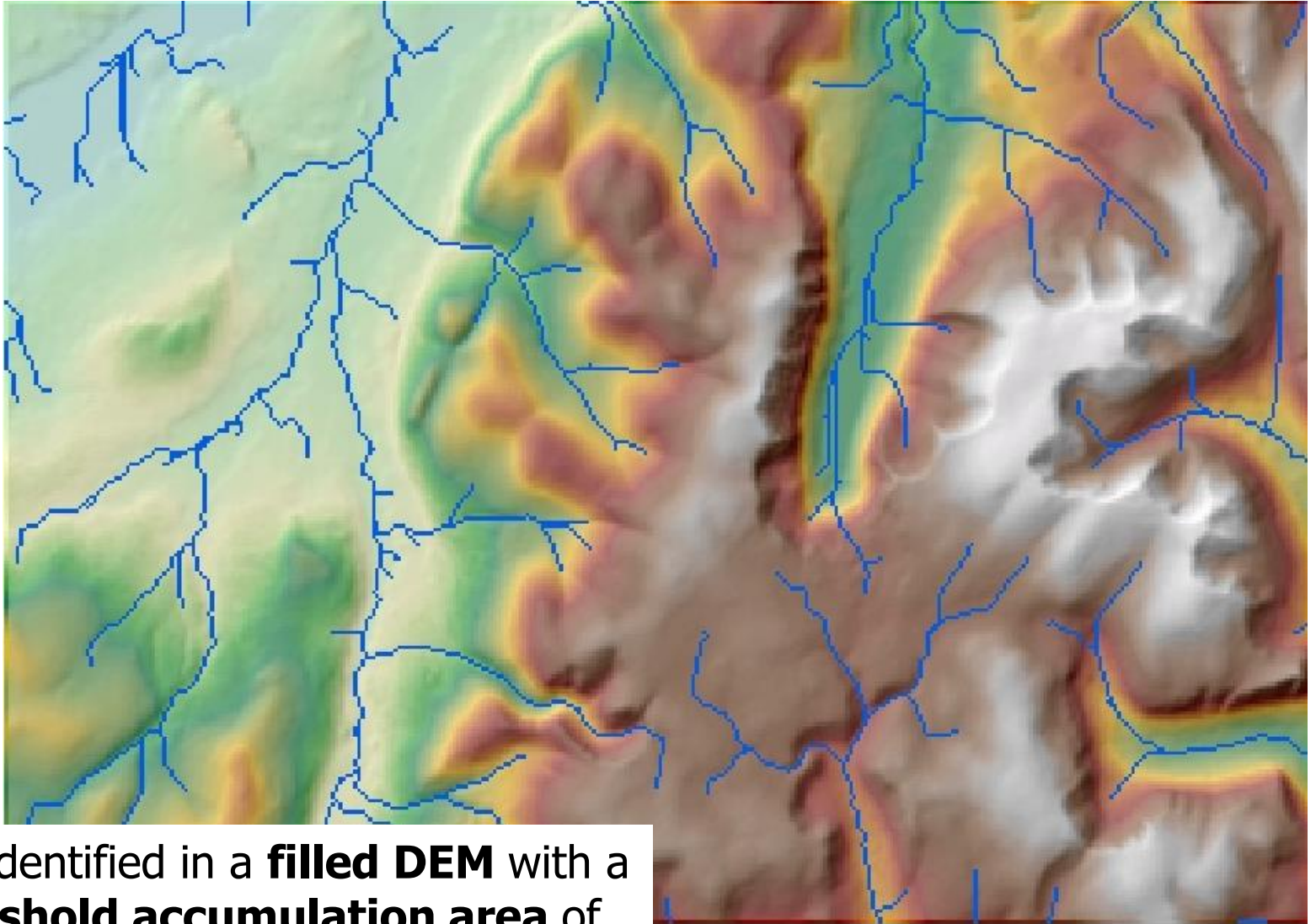


Source: [ArcMap help](#)

Identifying channels

- Several approaches to identifying channels
 - Use of a **threshold to identify channels** (e.g. cells with a flow accumulation greater than some specific value are flagged as streams)
 - This threshold may be **purely empirical** (does the stream network look realistic) or based on some knowledge of hydrologic properties (see later)...
 - A second grid can be overlain on the flow accumulation grid to simulate **input precipitation**
 - If we assume all this precipitation is **overland flow** then channel initiation can be calculated as some threshold of **accumulated water**...

Channel example



Channels identified in a **filled DEM** with a simple **threshold accumulation area** of 250 cells.

Resolution and DEM data quality

- Terrain models come in many **different resolutions** and are created from different datasets
- Common resolutions include 2m, 25m (Switzerland) 10 and 50m (UK), 30m (USA)
- Datasets with larger coverage at lower resolutions include ¹**GTOPO30** – a 1km dataset of the globe produced by the USGS, and **SRTM90**, a *near* global dataset at 90m resolution
- DEM data quality can also vary across products and space

¹ A free global dataset available online at <http://edcdaac.usgs.gov/gtopo30/gtopo30.html>

Scale and terrain...

The scale at which we measure a terrain has implications for the features we can represent/ extract...



More about resolution

- At the simplest level reducing resolution usually reduces the **maximum value** of gradients obtained and **decreases the smoothness** of the steps between slopes of different aspects
- At a more complex level information about **fine scale structure** of the landscape is lost
- We should be aware of the **purpose of measurements of slope** and consider sensitivity to resolution
- We often **can't control the algorithm**
 - we can often **choose a resolution**

DEM quality and networks

- McMaster refers to a result suggesting:

“ratio of **average elevation change** per pixel to **average elevation error** greater than one”

for

“reasonably accurate hydrological network”

- Thus in low elevation areas absolute error must be less (obvious!?)
- This result implies we should **divide** DEMs into areas where this criteria is met...

Applications of terrain data

Three examples of the use of terrain data:

- 1. Compound topographic indices** (use elements derived direction from DEM to generate physically meaningful property)
 - Stream power
 - Wetness index
- 2. Internal drainage** in Tibet
- 3. Slope algorithms, energy balance and penitentes**

Erosion processes

- **Stream power** can be defined as

$$\Omega = \rho g q \tan \beta$$

where

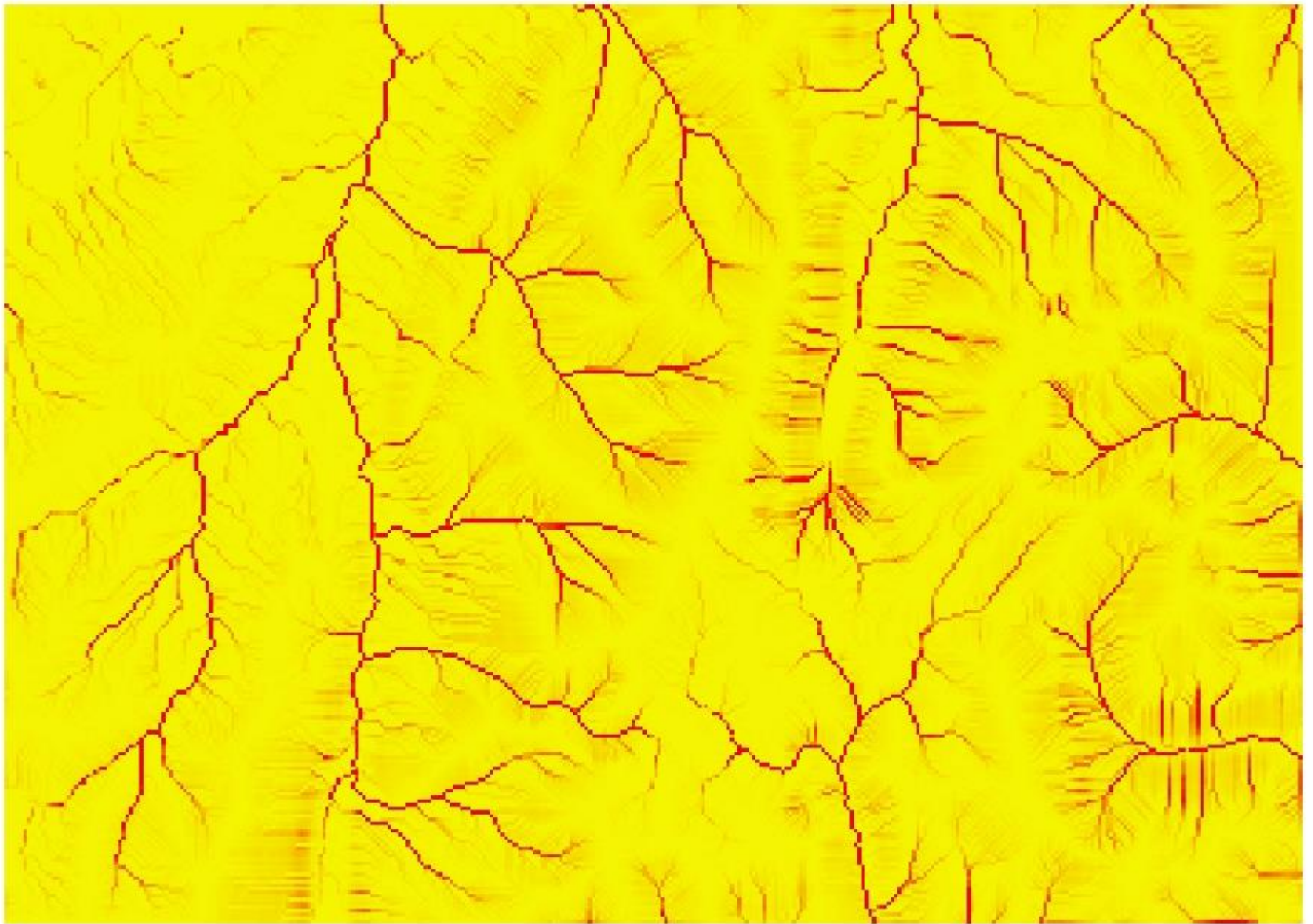
ρ is the density of water;

g is the acceleration due to gravity;

β is gradient; and

q the discharge per unit width.

- A_s (*specific catchment area*) is often considered to be **proportional to discharge**
- A_s defined as the upslope area per unit length of contour (or per pixel – the accumulation area)
- **Compound index** for stream power is thus
(ρ and g constant): $A_s \tan \beta$



Stream power index

Where is stream power high?

Wetness index

$$w_T = \ln\left(\frac{A_s}{T \tan \beta}\right) \text{ or } w = \ln\left(\frac{A_s}{\tan \beta}\right)$$

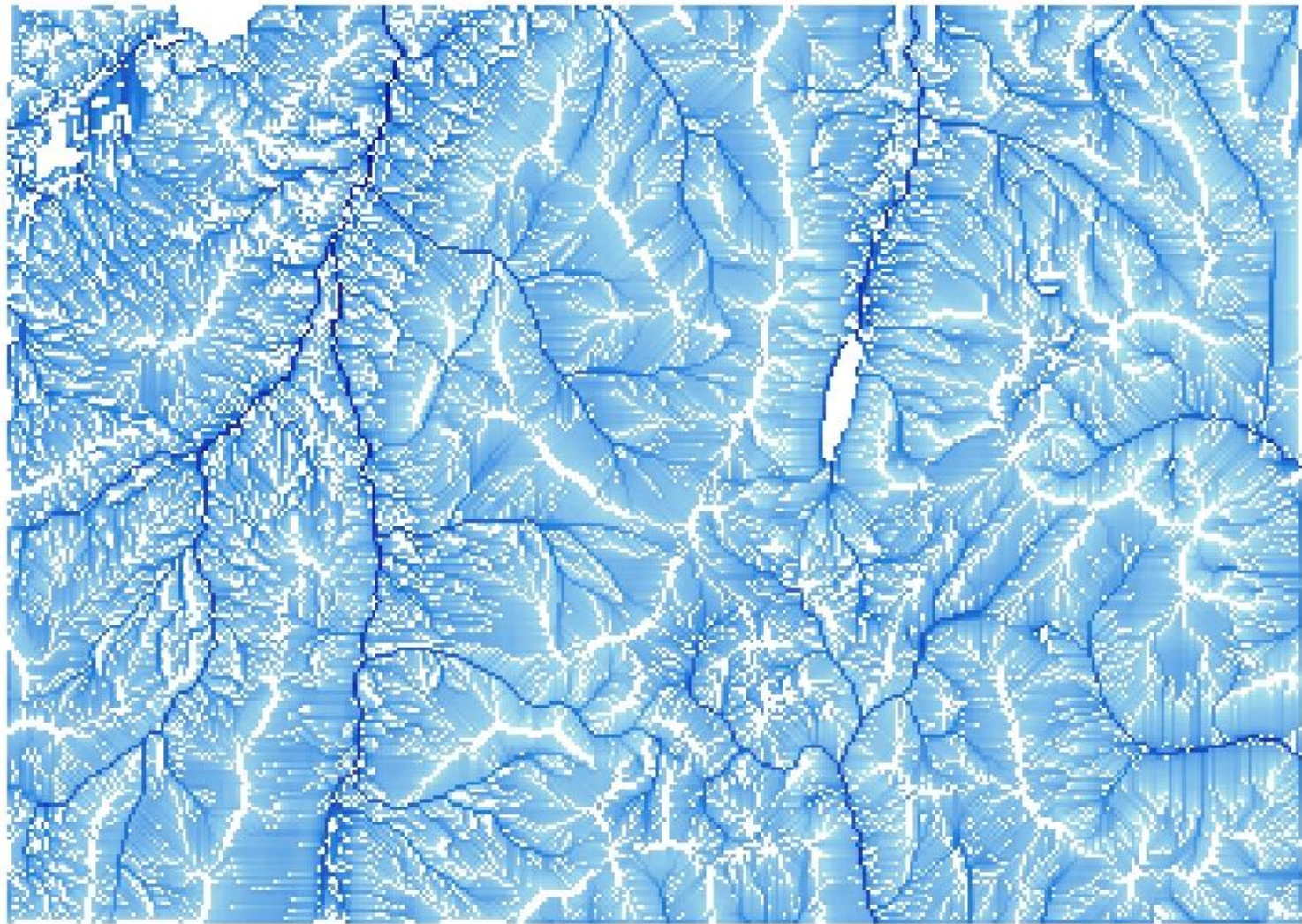
Here:

A_s is the **specific catchment area**;

β is the **gradient**; and

T is the **transmissivity** of a saturated soil profile.

If we assume the soil to be **homogenous** over the catchment then the second form is used ($T=1$)

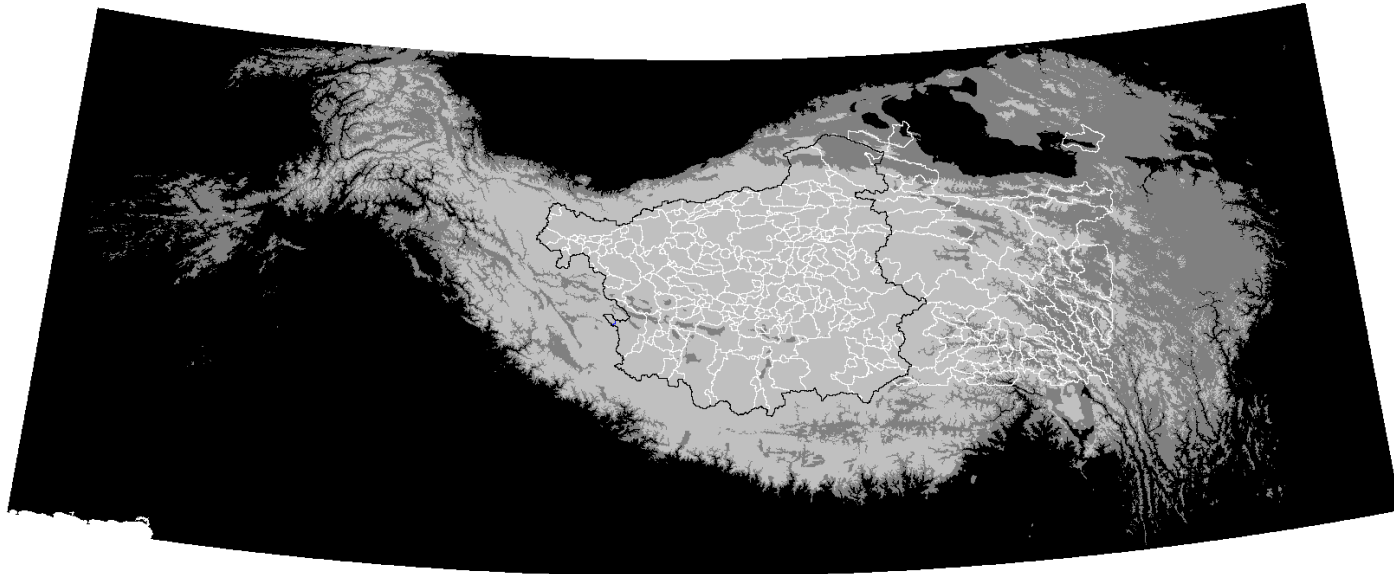


Wetness index...

Interpreting wetness index?

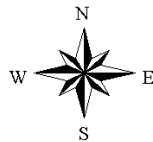
- **Correlation** found between w (wetness index) and **distribution of surface soil water content** in a small fallow catchment
- Indices using the product of **plan curvature** (rate of change of aspect) and **aspect** also found to give good correlations with surface soil water
- What **physical reasons** might there be for these correlations?

Studying internal drainage in Tibet



- Internal Drainage Divide
- < 3000 metres
- 3000 - 4500 metres
- > 4500 metres

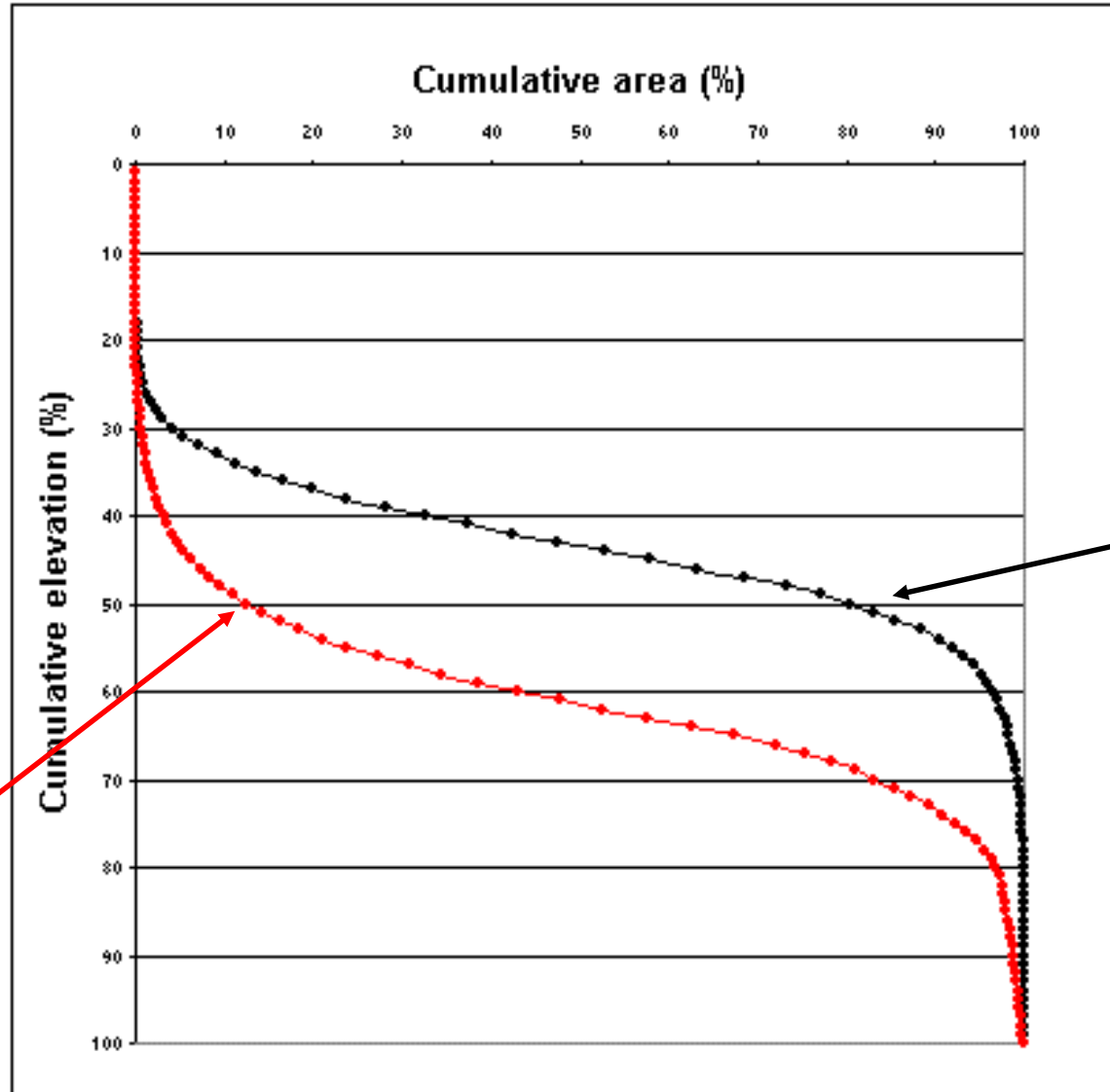
500 0 500 Kilometers



Projection: Lambert Conformal Conic
1st Standard Parallel: 20 Degrees North
2nd Standard Parallel: 35 Degrees North
Central Meridian: 84 Degrees East
Latitude of Projections Origin: 25 Degrees North
False Easting: 0.0
False Northing: 0.0

Tibetan plateau is 3500 x 1500 km. More than 82% is higher than 4000m. Half of the area is **internally drained** (a huge sink).

Hypsometric curves



80% of the area,
upper 50%
of elevation
– **externally
drained**

12% of the area,
upper 50%
of elevation
– **internally
drained**

Internal drainage in Tibet

- **Hypothesis** – morphometric parameters are statistically different between internally and externally drained areas
- Data at a resolution of **1km**
- Values of **gradient** far from 'real'
- Results showed difference between mean gradient (**2.6°**) in internally drained with externally drained areas (**6.7°**)
- Internally drained area shown by **hypsometry** to be very dissected relative to externally drained area
- This work possible at **low resolutions**

Modelling energy balance in the Andes (1)

- Modelling energy balance for the formation of **penitentes** in the Andes
- Interested in **thresholds** for their formation – so **absolute values** of energy are very important
- **Direct** and **reflected** radiation both very important – both are directly related to **terrain** (and **slope**)
- Corripio argued that 3x3 cell algorithms **oversmooth** extreme values (important since interested in process thresholds)

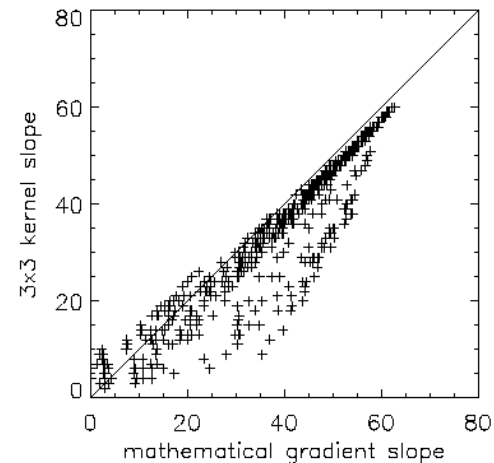
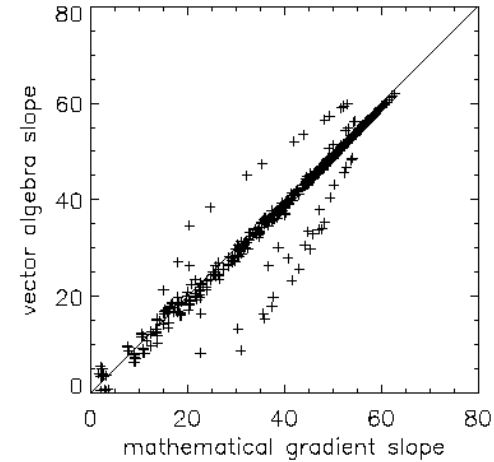
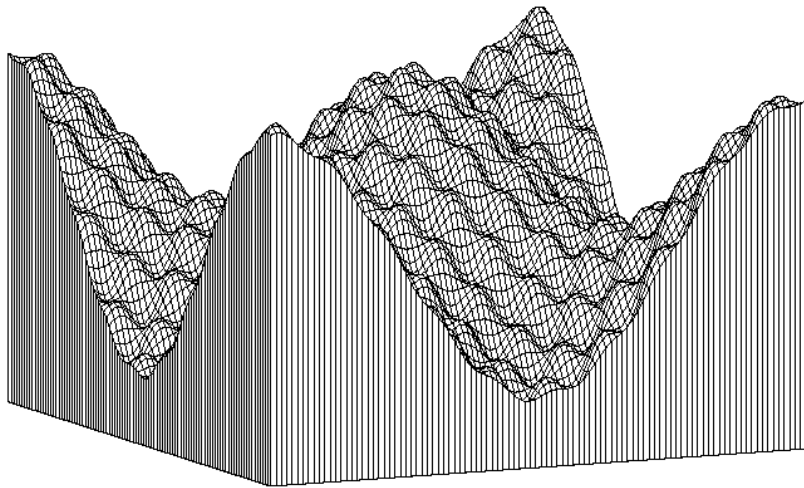


<http://www.geo.ed.ac.uk/~hasas/pttfield.html>

Modelling energy balance in the Andes (2)

- Corripio's algorithm calculates the **normal** to the **average** of the **two planes** enclosed by 4 points/ cells using vector algebra
- He **compares** his results to a **synthetic surface** with an **analytical solution** and **ArcGIS's** gradient calculation

Algorithm	Spearman's rank correlation	rms $\hat{\epsilon}$	σ	mean abs. dev.
math. gradient	—	—	—	—
vector algebra	0.993804	0.7089	1.7993	0.8205
3x3 kernel	0.980826	1.2430	3.2623	1.7720



Synthetically generated surface

$$z = \cos x \cos y + a \sin(bx) \sin(by)$$

Summary

- **Surfaces** (and **representation**) matter – **terrain** is central to many problems
- GIS implement a **multitude of algorithms** – remember ways in which uncertainty can be introduced
 - **Concepts** (e.g. raster/ TIN)
 - **Measurement and representation** (Data uncertainty)
 - **Analysis** (Algorithms and models)
- Hydrological modelling has multiple steps where uncertainty can be introduced
- Three examples of the use of terrain data to explore complex questions

Next week

- We will look at **visibility analysis** – what is it and how can we use it?
- We will examine the basic concepts of **viewshed algorithms**
- Is there **more than one way** to derive viewsheds and does it matter?
- How can viewsheds be **applied** in spatial analysis

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More information

- Many of the figures were produced using Landserf which you can get at
<http://www.soi.city.ac.uk/~jwo/landserf/>
- GITTA materials are available in the basic level on Terrain Analysis